

# Linking Glacier Erosion, Mass Balance and Climate

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## Abstract

The distribution of surface area of a glacier with altitude (its AA profile) developed over the thousands or millions of years of its existence. The AA profile for each glacier is unique and reflects the interaction of the regional climate with the bedrock topography on which the glacier rests. With the passage of time, erosion produced by the sliding glacier carves into the underlying rock a distinctive bed that is reflected by its AA distribution. The AA profile can then be thought of as a weather recorder that integrates, by erosion processes, the long-term climate of the region in which the glacier exists. A recently developed mass balance model (the PTAA model) has the underlying premise that an inextricable link exists between climate, glacier erosion and the mass balance. It is because of this link that the long-term mass balance of a glacier and its distribution with altitude can be determined using only observations from low altitude weather stations and the area-altitude distribution. Thus, by applying available meteorological observations, namely precipitation and temperature, from usually much lower altitude stations, plus the distribution of a glacier's area as a function of altitude, the mass balance distribution with altitude can be determined for any alpine glacier for which these data are available. To verify the model's accuracy and reliability, it is applied to South Cascade Glacier, Washington, which has a long record of measured mass balance. Comparisons with South Cascade Glacier show only fair agreement between measured and simulated annual balances ( $R^2 = 0.57$ ), however, the simulated volume change over a 22 year period demonstrates excellent agreement with the geodetic balance, which was calculated from maps made of the glacier in 1975 and 1996.

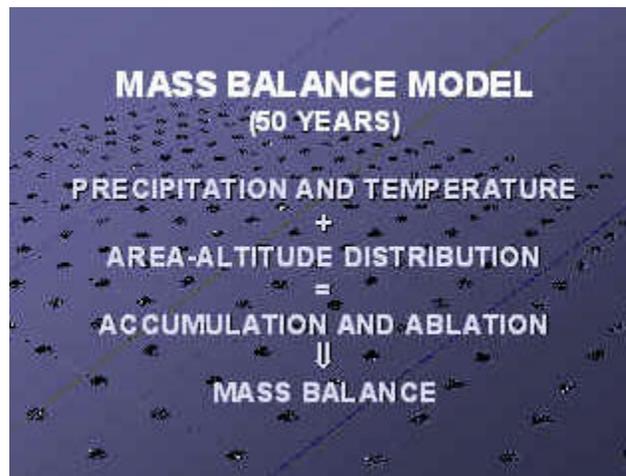
## Introduction

In the thousands or even millions of years of a glacier's existence the following sequence has occurred:



**Figure 1. The climate/area-altitude distribution link is controlled by erosion.**

By slightly revising the sequence shown in Figure 1, a mass balance model can be developed that uses only easily collected meteorological variables (which represent the climate) and the area altitude distribution (calculated from a topographic map of the glacier). Figure 2 demonstrates the basis for a predictive mass balance model.

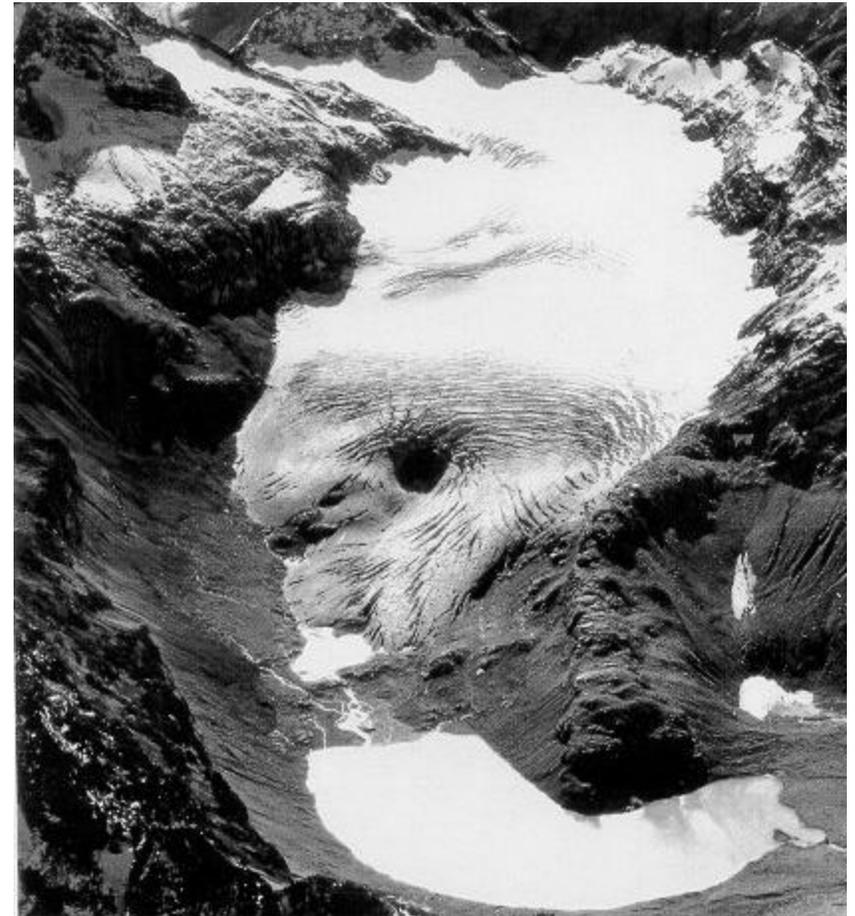


**Figure 2. For developing a balance model with available data, the climate is represented by daily precipitation and temperature observed at nearby but much lower altitude weather stations.**

The model is tested for South Cascade Glacier, located in the North Cascade Range of Washington State. The glacier has been studied since 1957 and has a 40-year history of continuous mass balance measurements. The glacier has been losing mass at an average rate of approximately 1 meter per year during this period. Photographs taken in 1960 and 1992 (Figures 3 and 4) demonstrate the large loss in mass this glacier has undergone.

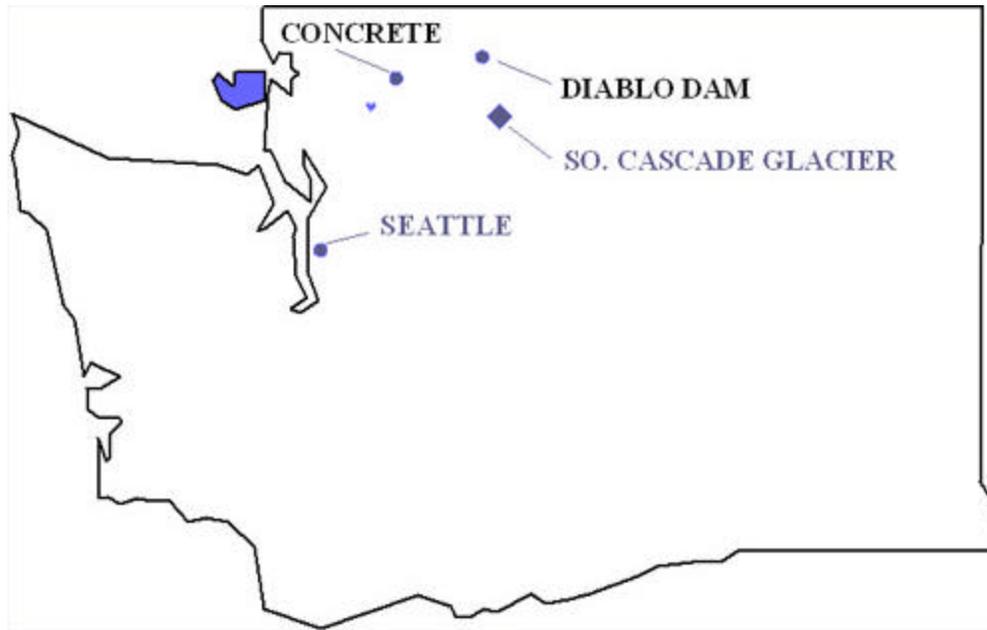


**Figure 3. South Cascade Glacier on September, 29, 1960. Photo by Austin Post. The glacier is approximately 0.8 km wide at the ELA and 3.4 km long.**



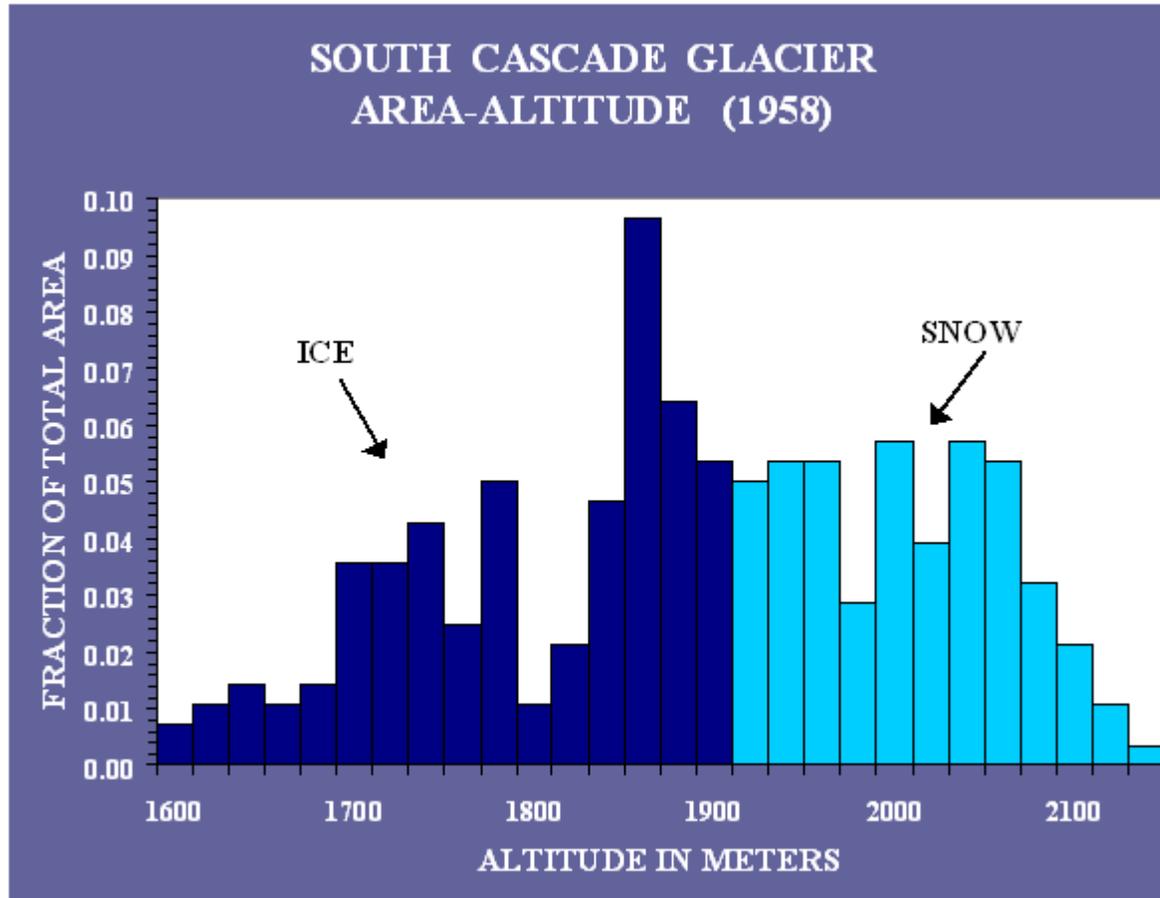
**Figure 4. South Cascade Glacier on September 9, 1992. Photo Robert Krimmel, US Geological Survey. The glacier lost 30 m of mass during the 32 year period between 1960 and 1992.**

The weather stations used to represent the climate at South Cascade Glacier over the past 65 years are at Concrete and Diablo Dam. Their locations in Washington State with respect to the glacier are shown in Figure 5.



**Figure 5. Map of Washington State showing locations of South Cascade Glacier and the two weather stations used in this study to represent the glacier's climate.**

The area-altitude distribution of this glacier used to calculate balance variables is shown in Figure 6.



**Figure 6. South Cascade Glacier surface area for each 20 meter altitude interval (in 1958). The fundamental premise of the mass balance model is that this distinctive profile developed over thousands of years by climate-driven erosion caused by ice abrasion of the underlying bedrock.**

### Simulation Model Structure

A total of fourteen coefficients are used in algorithms that convert daily meteorological observations, collected at low-altitude stations, to

snow accumulation and snow and ice ablation at the glacier. Eight balance variables are then calculated daily from these data for as long a period as weather records are available.

The balance variables determined daily for each 20 meter altitude increment on the glacier are:

1. Snow accumulation or rain
2. Ablation
3. Mass balance
4. Runoff
5. Mass Exchange

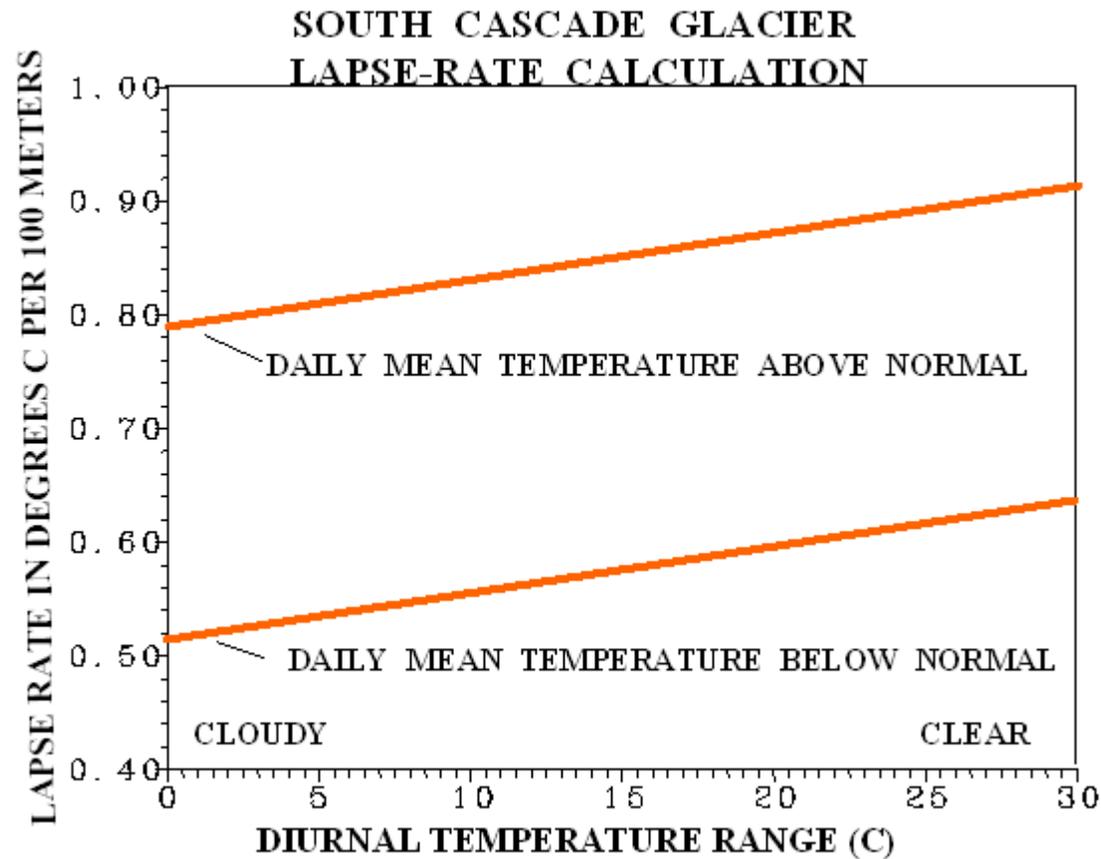
Based on these results, the following are then determined for each day of the ablation season, from approximately June 1 to September 30:

6. Accumulation Area Ratio
7. Snowline Altitude
8. Zero Balance Altitude

### ***Temperature Lapse-Rate***

A critical task in determining these eight variables is the lapse-rate of the temperature observed at the valley stations, to the glacier, which for South Cascade Glacier is at an altitude 1300 meters higher than the temperature station. Four coefficients, which are determined by calibration, are required to calculate the daily lapse-rate (Figure 7).

A cloudy day (a low temperature range) with below normal temperatures has the lowest lapse-rate (approximately 0.54 degrees C per 100 meters), and a day with clear skies (a high temperature range) and above normal temperatures has the highest lapse-rate (about 0.90 degrees per 100 meters).



**Figure 7.** The temperature lapse-rate is dependent on two factors: the deviation of the mean daily temperature from the daily average and the diurnal range in temperature (daily maximum minus minimum), which is an cloudiness index. Two coefficients determine the intercepts of the lines that relate lapse-rate to temperature range, and two coefficients are the slopes of these lines.

### *Precipitation*

Daily precipitation observed at the two weather stations is converted to precipitation at each of the twenty-eight 20-meter altitude intervals,

using 3 coefficients; one determines the altitude of maximum precipitation, one is a multiplier (times measured precipitation) that determines precipitation at maximum altitude, and one is a multiplier for precipitation at the terminus.

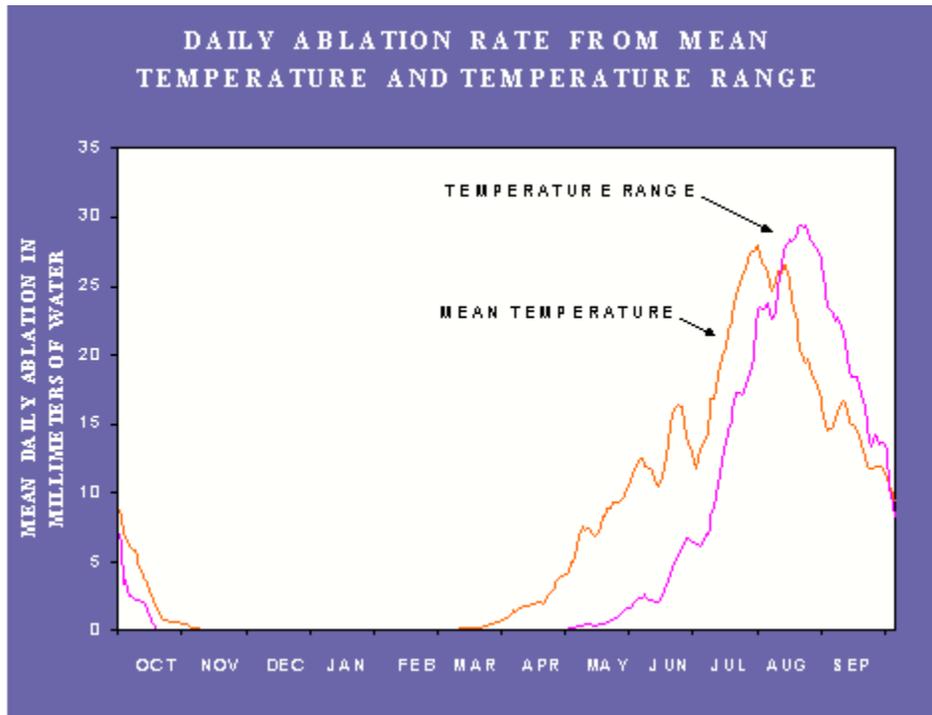
### ***Accumulation***

Snow accumulation at each altitude interval is determined by calculated precipitation and temperature. If the daily mean temperature is less than zero, precipitation occurs as snow; if temperature is equal to or greater than zero, precipitation occurs as rain. Beginning on October 1, snowfall is cumulated at each altitude interval until the following September 30. Rain is assumed to occur as direct runoff.

### ***Ablation***

Four coefficients are required to calculate snow and ice ablation. Two different algorithms are applied: one for snow and ice, and one for only ice ablation. Both the mean daily temperature and the diurnal temperature range are used in these algorithms.

For only ice ablation, a short wave, radiative ablation factor is calculated from the diurnal temperature range, which is a rough measure of cloudiness, or solar radiation.



**Figure 8.** Ablation is calculated from both the mean daily temperatures (which represents turbulent heat transfer, orange line) and the daily temperature range (which represents solar radiation, pink line). Total ablation is the sum of these two components.

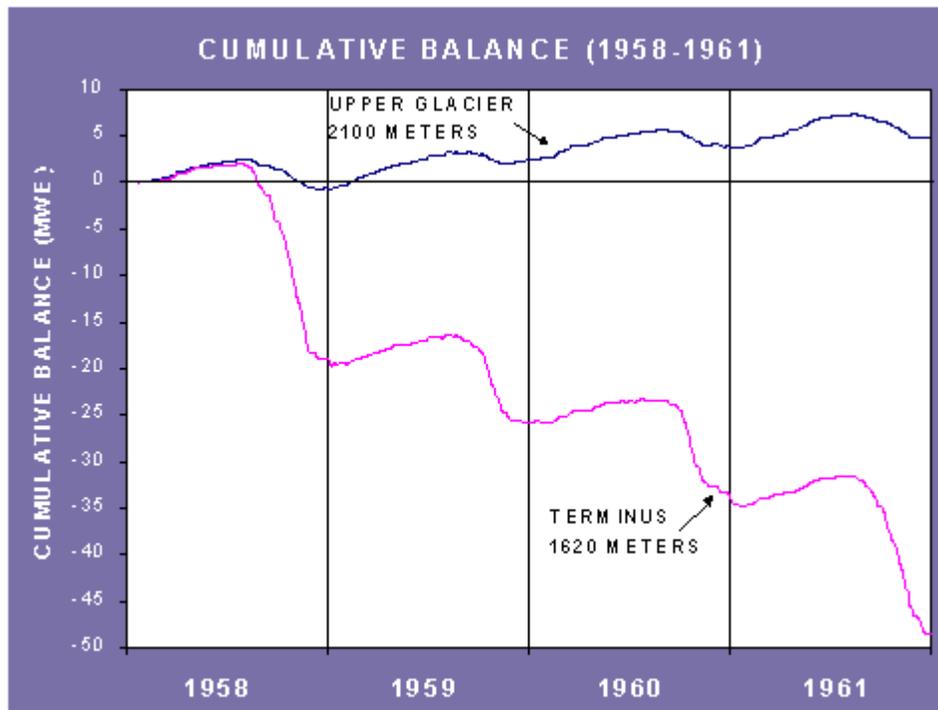
### *Balance*

Each altitude interval is treated as a separate entity and its mass balance is independently calculated. By applying temperature lapse-rate and precipitation coefficients, the following variables are calculated for each altitude interval,  $z$ , and for each day,  $i$ :

1. Precipitation as snow or as rain,  $c(z,i)$  and  $pr(z,i)$
2. Ablation of snow and ice,  $a(z,i)$

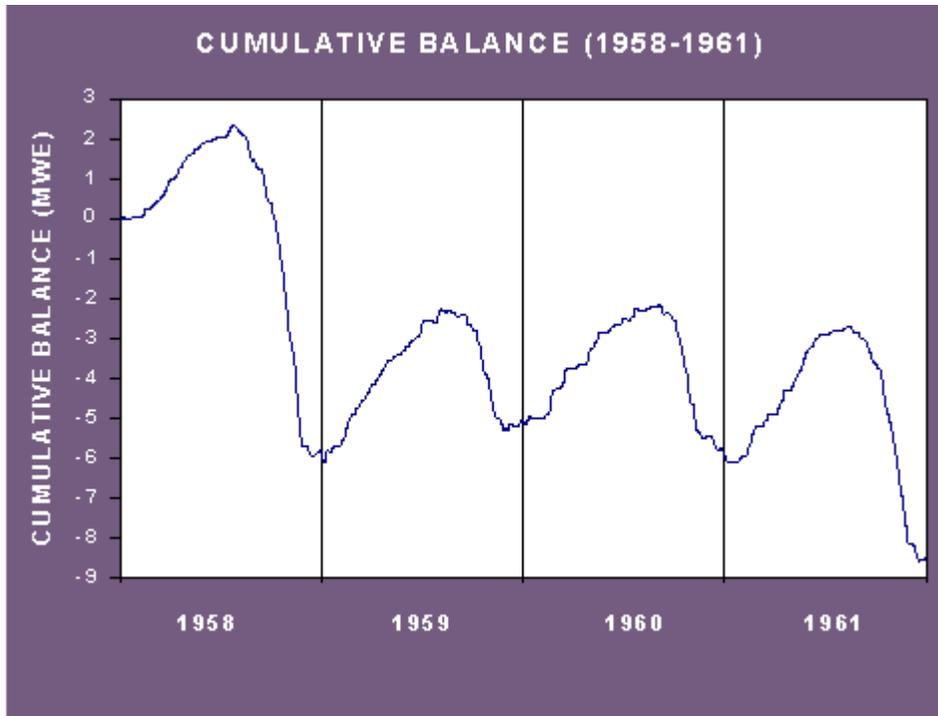
### 3. The mass balance, $b(z,i)$

The continuous balance at the terminus (1620 m) and the head of the glacier (2100 m) for the 1958 to 61 period is shown in Figure 9.



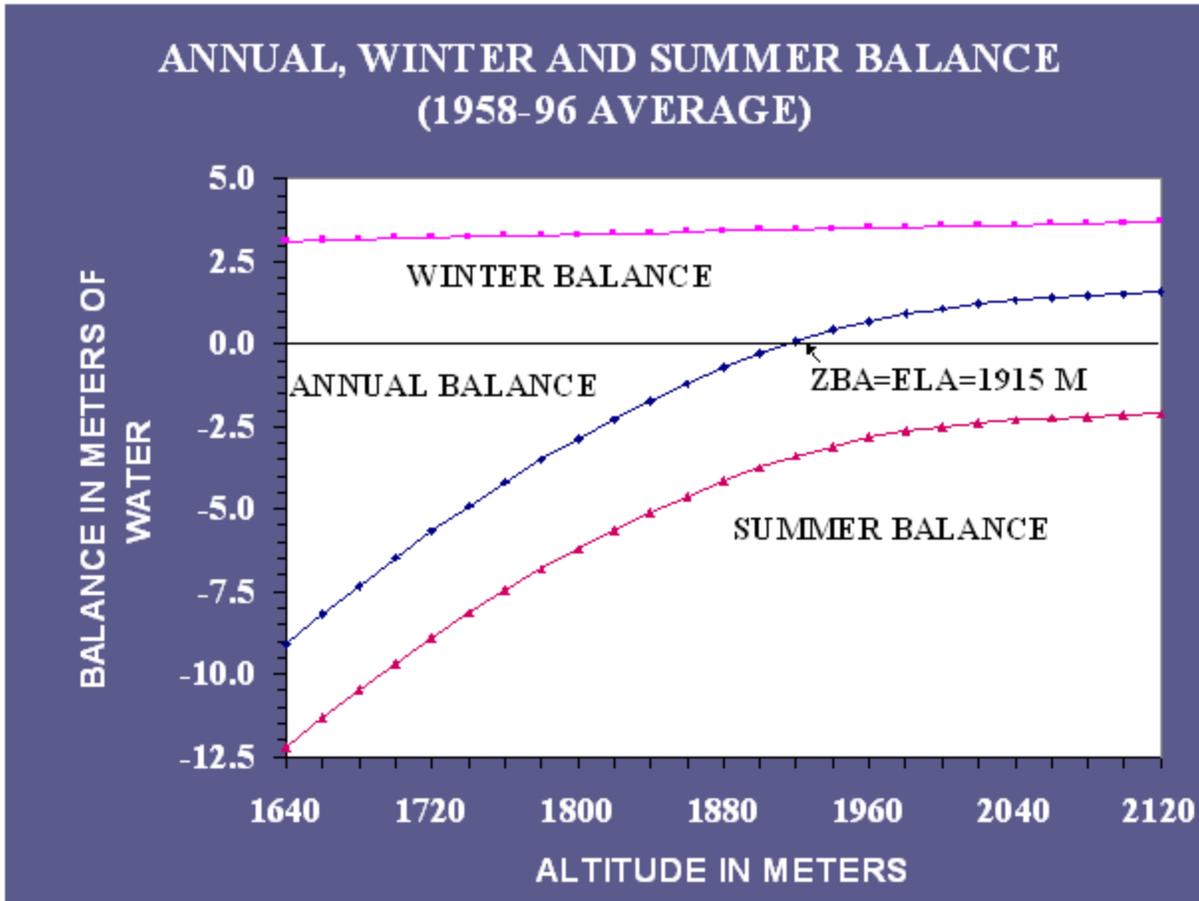
**Figure 9. Cumulative daily balance ( $b(i,z)$ ) for four years at the terminus and at the head of the glacier demonstrates the moderate gain in mass in the accumulation area and great loss in mass in the ablation area.**

The continuous daily balance,  $B(z)$ , determined by integrating the balance and area for the total glacier, is shown for the 1958-61 period in Figure 10.



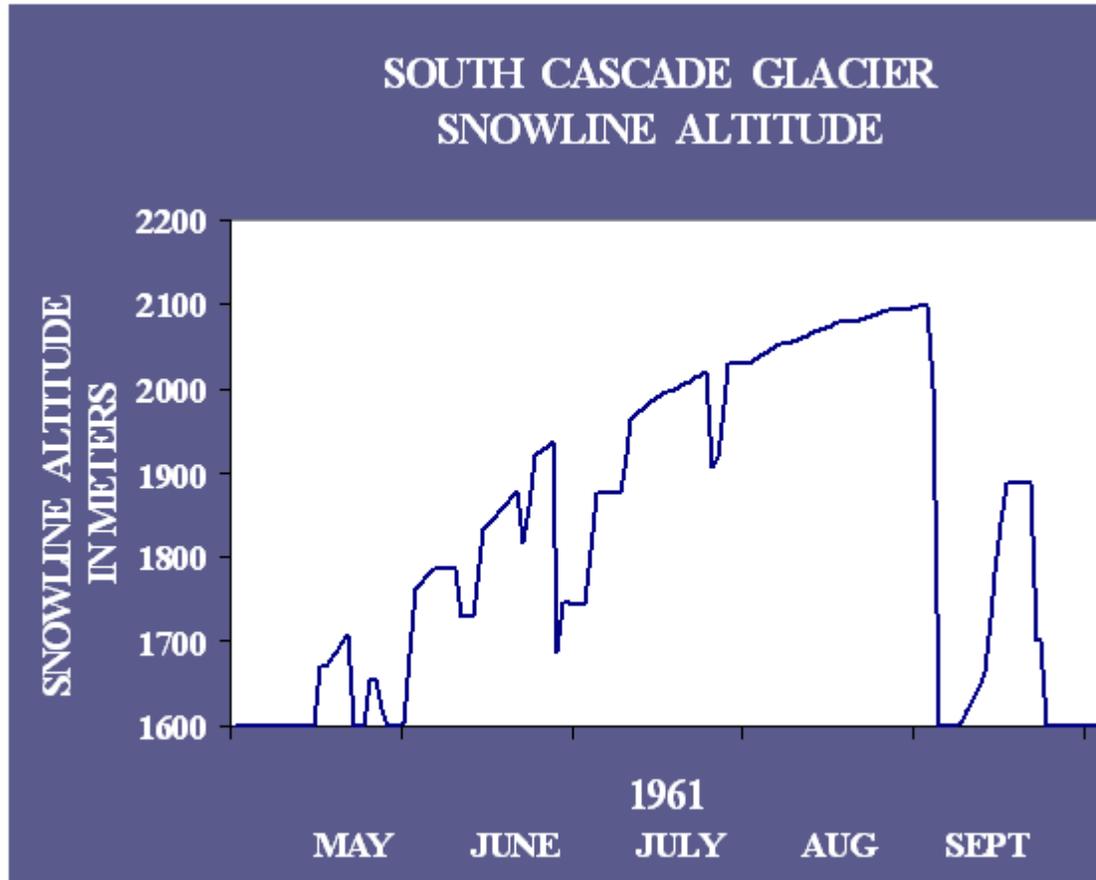
**Figure 10.** The cumulative glacier balance for the 1958-61 period demonstrates the variation in mass of this glacier each year.

The altitude distribution of the winter, summer and annual balance can now be calculated. Figure 11 shows the balance-altitude distributions averaged for the period of record.



**Figure 11.** The altitude distribution of the mean winter,  $c(z)$ , summer,  $a(z)$ , and annual balances,  $b(z)$ , averaged for the 1955-96 period. The interval for  $z$  is 20 meters. The zero balance altitude (ZBA) on September 30 is approximately the ELA (1930 m).

The simulated daily snowline altitude for May 1-September 30, 1961 is shown in Figure 12.



**Figure 12.** The simulated daily snowline altitude, May 1-September 30 1961. Note that the rise in temporary snowline altitude after a summer storm is much more rapid than that of the seasonal snowline. An unusual late summer storm on September 1 caused the snowline to drop well below the glacier and essentially ended the ablation season that year.

### *Zero Balance Altitude*

The zero balance altitude is calculated daily from the time the first ice is exposed at the terminus until the glacier is again covered by snow the

following winter. The summer balance season ranges from 59 days (beginning on August 2) to 182 days (beginning on April 1) and averages 129 days (corresponding to a May 24 start date for the summer season).

### ***Mass Exchange***

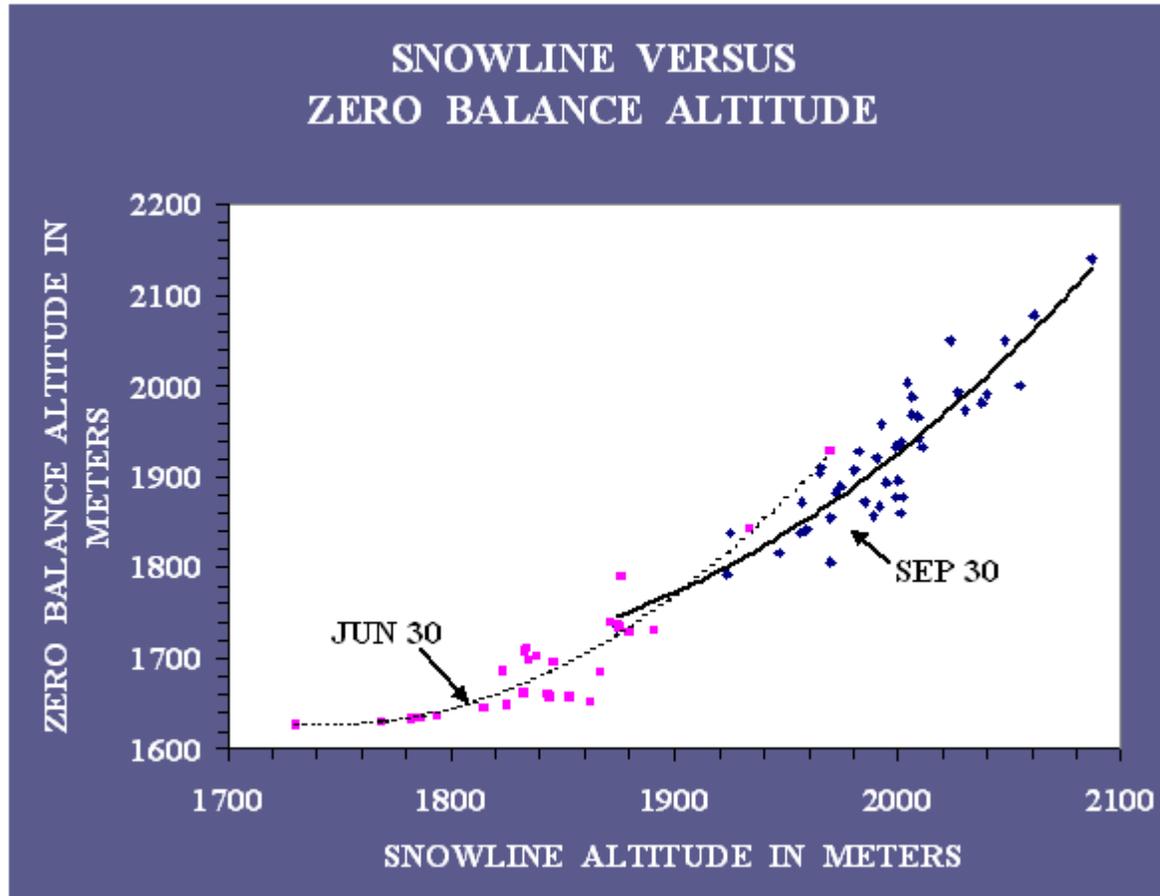
The movement of ice mass from higher to lower altitudes of a glacier due to gravity produces erosion and ultimately determines the altitude distribution of glacier area. The exchange of mass that occurs across the line of zero balance (ZBA) is then considered to be an effective means to relate erosion to mass balance.

### ***Accumulation Area Ratio***

The fraction of the area above the ZBA to total glacier area is defined as the AAR and is calculated daily from the first day that ice becomes exposed at the terminus, or the start of the summer ablation season, through September.

### **Calibration**

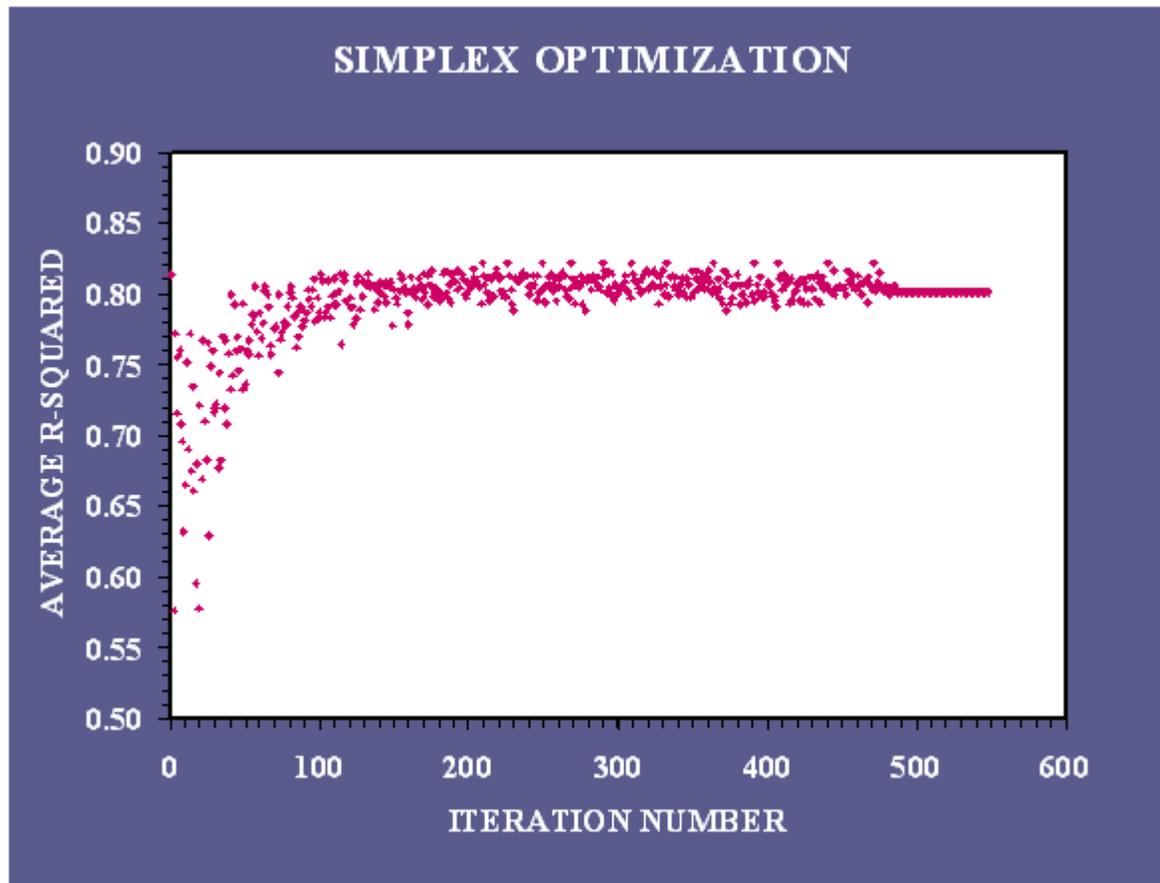
Optimum values for each of the 14 coefficients are solved for simultaneously using a simplex parameter optimizing procedure. The objective error that is minimized in the simplex is the complement of the explained variance ( $1-R^2$ ) from a polynomial regression fit of several balance variables that are determined each day during the ablation season. For example, the balance for the total glacier is regressed against the zero balance altitude for each day from approximately June 15 to September 30 (107 days). Two days (June 30 and September 30) of this 2-degree polynomial regression are shown in Figure 13.



**Figure 13. Glacier balance as a function of the zero balance altitude for June 30 and September 30 (each point represents one year of the 1955-96 period). The  $R^2$  for a 2-degree polynomial fit for June 30 is 0.85 and is 0.96 for September 30.**

Nine different regression pairs are used for a total of 963 regressions (if June 15 and September 30 are the start and ending days). An average of the explained variance complement ( $1-R^2$ ) for all regressions is then minimized in the simplex to obtain optimum coefficient values.

The results for determining optimum coefficient values from the maximum average  $R^2$  are demonstrated in Figure 14. The  $R^2$  produced after 548 iterations is 0.801, which is an average of 657 polynomial regressions for each iteration. The basis for using a statistical verification technique is that an internal consistency is thought to exist among the various parameters that are generated by combining meteorological data with the glacier's area altitude profile. Obviously, for this approach to work, the meteorological records must be from stations that are close enough to sample the same air masses and weather conditions as the glacier.



**Figure 14.** Results of the simplex optimization process to obtain the maximum  $R^2$ , which is an average of 657 2-degree polynomial regressions. There are 548 iterations and the  $R^2$

average is 0.801.

### **Comparison with Measured Balance**

A rigorous test of the model's accuracy and reliability is by direct comparison with measured balances produced by South Cascade Glacier field surveys. Continuous mass balance data have been collected at this glacier since 1959. These measurements, which are labor intensive and expensive, are considered the most accurate method yet devised for determining a glacier's annual mass balance.

A year-by-year comparison of simulated and measured balances suggests that there are substantial errors in one or both methods. Figure 15 is a plot of annual measured versus simulated balances for the 1959-96 period. The explained variance fraction for a linear regression fit is 0.57 with a standard error of 0.75 meters (we).

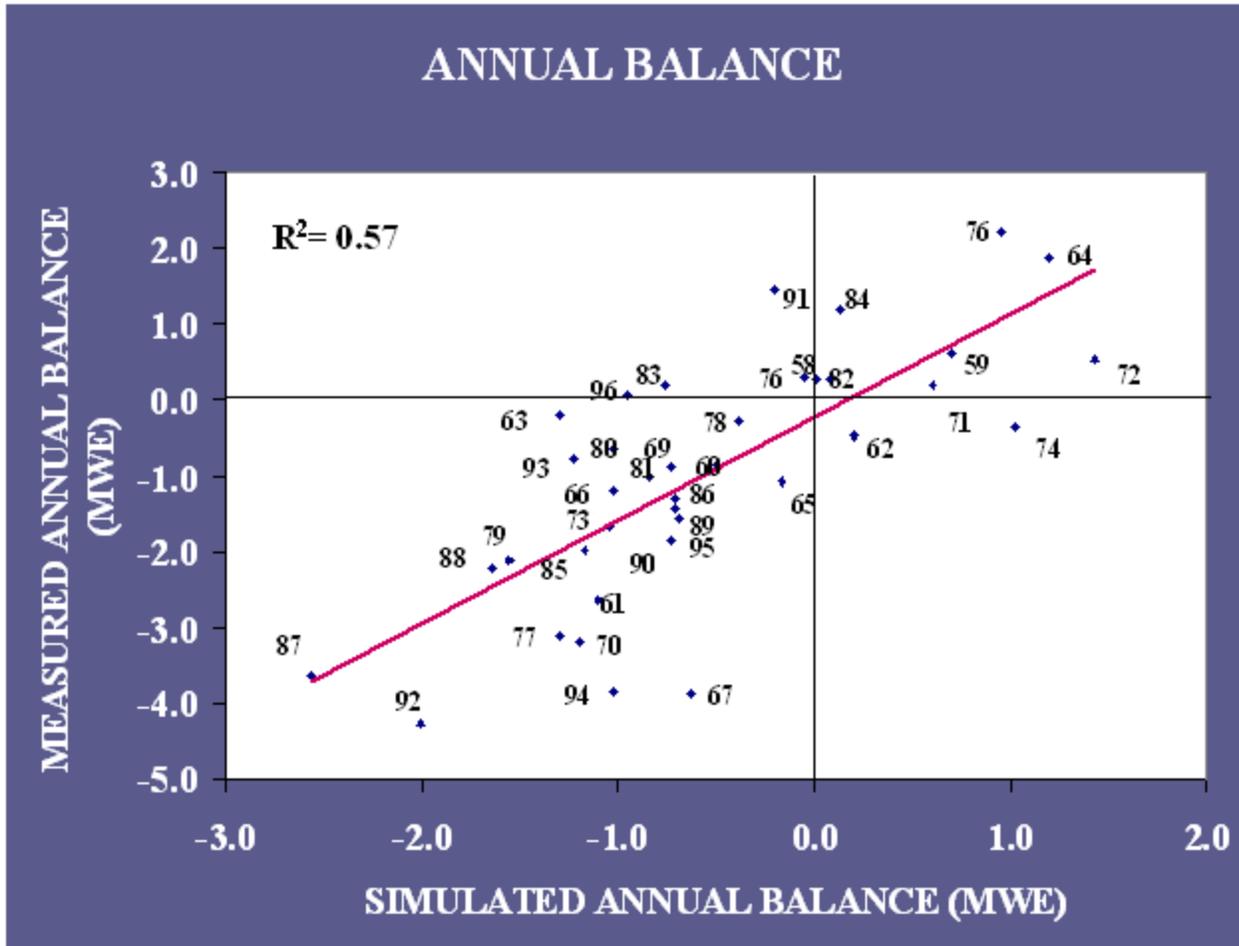
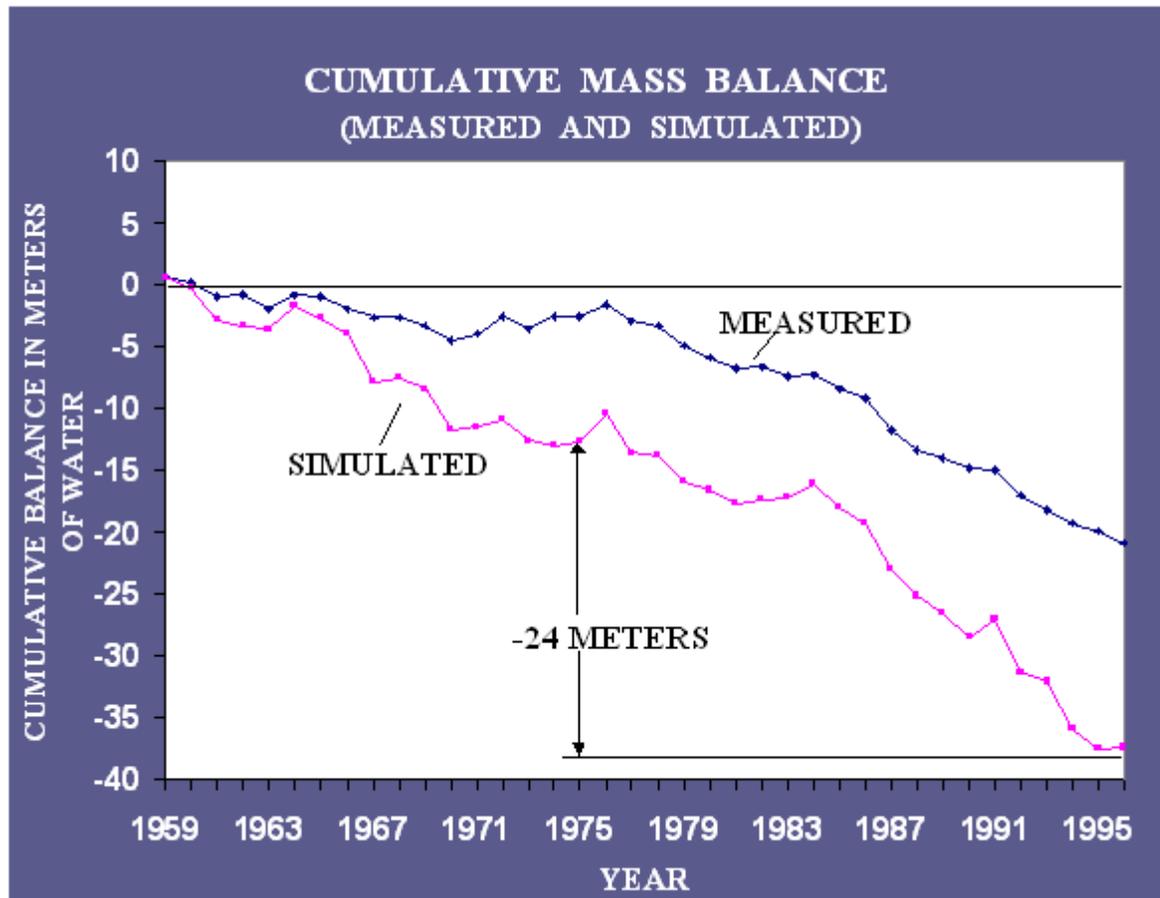


Figure 15. The measured versus simulated annual balance for each year of the 1959-96 period. The  $R^2$  for a linear regression is 0.57 and the mean root square error of simulated balance is +/- 0.75 m.

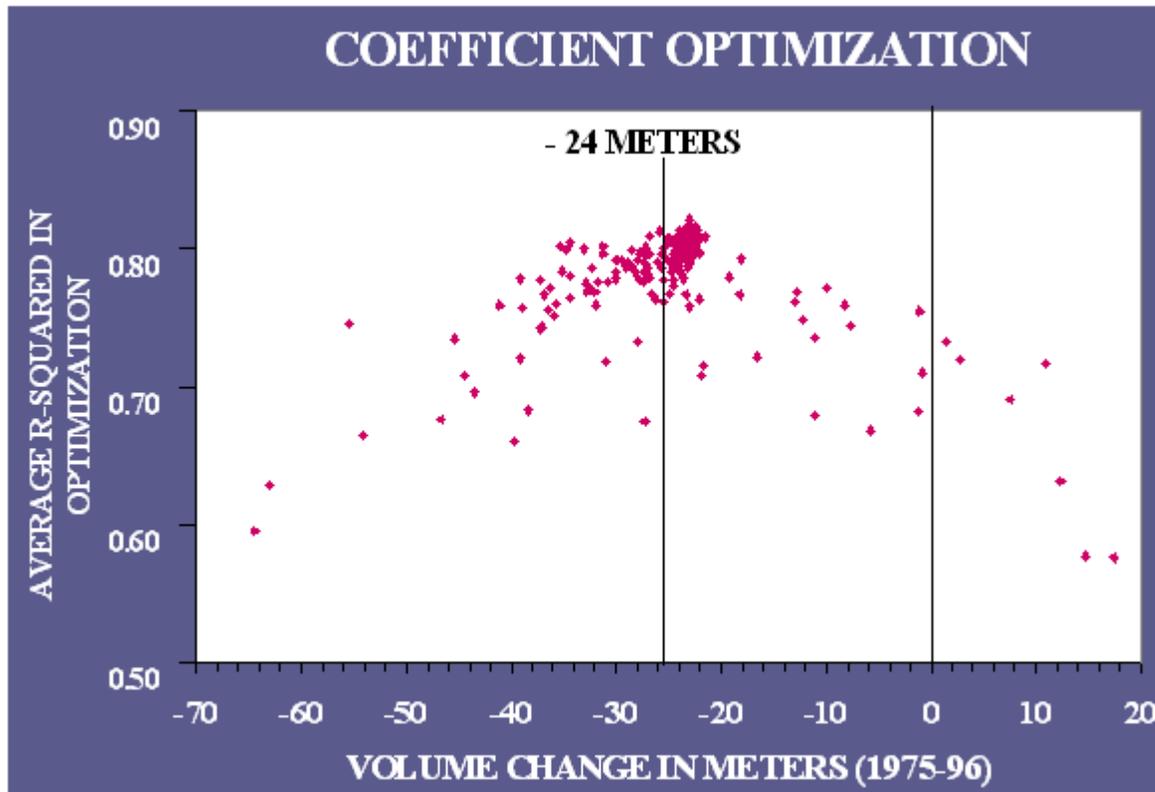
A plot of cumulative balances (measured and simulated) in Figure 16 indicates that the simulated balances is consistently more negative than measured. However, the geodetic method used for calculating the volume change between 1975 and 1996 equaled -24 meters, compared with

a direct measurement of annual balances of  $-18$  meters.



**Figure 16. Cumulative simulated and measured balance for the 1959-96 period. The 1975-96 measured volume change is  $-18$  m (we); the simulated loss for the same period is  $-24$  m (we), which is equal to the geodetic balance measured for this period.**

The model's simulated volume change for this period was also  $-24$  meters, indicating that the model and geodetic results are in good agreement. A plot of simulated volume change versus the mean  $R^2$  determined by coefficient optimization indicates that the  $-24$  meter volume change is valid and real (Figure 17).



**Figure 17.** The relationship between the  $R^2$  for the iterations and the corresponding 1975-98 volume change calculated for each set of trial coefficients. The volume change when the maximum  $R^2$  was reached is approximately  $-24$  meters (we). Note that the maximum  $R^2$  determination is completely independent of the amount of volume change.

## **Conclusions**

Glacier mass balances and other related variables can be simulated with acceptable accuracy using only low-altitude weather observations (precipitation and temperature) and the area-altitude distribution of the glacier. It is postulated that this can be accomplished because erosion of the bedrock underlying the glacier has developed a surface configuration that has created a mass balance link between the climate and the area-altitude profile.

## **References**

Tangborn, W.V., A Mass Balance Model That Uses Low-altitude Meteorological Observations and the Area-Altitude Distribution of a Glacier, *Geografiska Annaler* - 81A, pp 753-765 (1999).